Technical Notes

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Measurements of Turbulent Flow Behind a Flat Plate Mounted Normal to the Wall

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Introduction

TURBULENT flows in streamwise corners have attracted many researchers[1-4] because of their importance in practical applications. Equally important "slender shear flows" are the flows in the region downstream of these streamwise corners. The interaction between the fuselage boundary layer and the wing wake is such a flow, and an important one in determining aircraft performance characteristics. However, there are no published studies on this type of turbulent flow. Investigations of longitudinal vortices imbedded in boundary layers by Tanaka and Suzuki[5] and Mehta et al.[6,7] have relevance, since the behavior of the longitudinal vortices generated in the corner play an important role in the development of such flows. The present Note presents the results of measurements made in the turbulent flow behind a thin plate mounted normal to a larger boundary-layer plate. This is an idealized wing-wake/boundary-layer interaction, but it may better be thought of as the merging of two coflowing 90-deg corner flows when the dividing wall terminates, since the leading edge of the plate simulating a wing is aligned with that of the wall simulating a body. The essential difference between this flow and that behind a wing on a body is that, in the former, the horseshoe vortex is absent and the secondary flow is represented by a pair of weak counterrotating vortices downstream of each corner, resulting in a total of four vortices.

Model and Measurement Methods

Measurements were made in the 2×2.5 ft low-speed wind tunnel at California State University, Long Beach. At the test speed of 36 m/s, the freestream turbulence was less than 0.4%. The flow configuration under investigation is depicted in Fig. 1. The models were made of two aluminum plates. The longer plate (the "body") had a few streamwise and spanwise arrays of pressure orifices. The aft 16 cm of the "wing" was tapered down to a trailing-edge thickness of less than 0.15 mm with a wedge angle of 3 deg. The leading edges of the two plates were aligned and trip wires were fixed to insure transition at 2.5 cm downstream of the leading edges. Two corner flows were formed at the foot of the wing. These corner flows merged in the region downstream of the wing's trailing edge and a wake developed downstream of it interacting with the continuing boundary layer on the body.

Measurements of the primary mean velocity component U were made using a round pitot tube of 0.33 mm o.d. in the region $0 \leq x \leq 406.4$ mm. Mean velocity components U and V, and five Reynolds stress components, $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$, $-\overline{uv}$, and $-\overline{uw}$, were measured by a two-channel TSI 1050 hot-wire anemometer with TSI 1243 X-wire probes. The hot-wire signals were analyzed by analog methods.

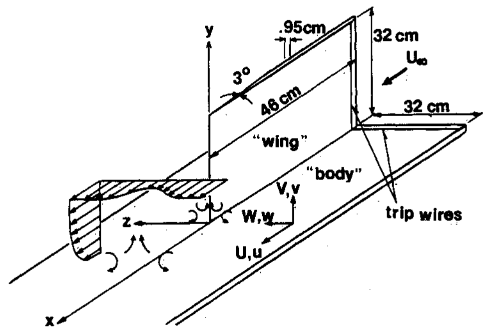

Fig. 1 Flow configuration.

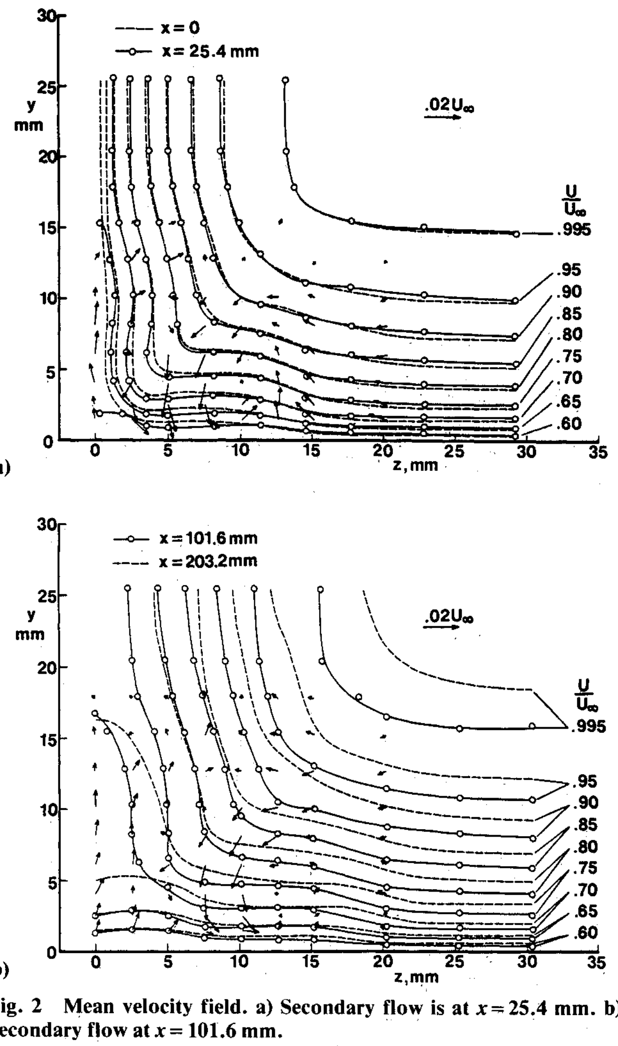

Fig. 2 Mean velocity field. a) Secondary flow is at $x = 25.4$ mm. b) Secondary flow at $x = 101.6$ mm.

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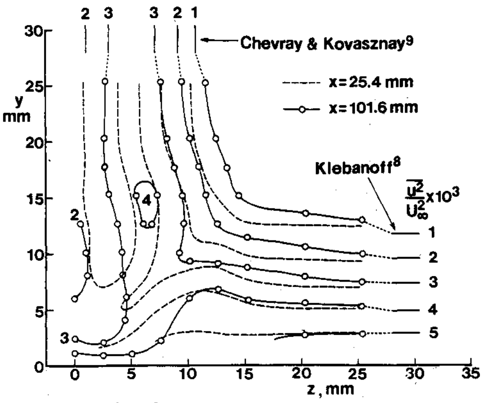
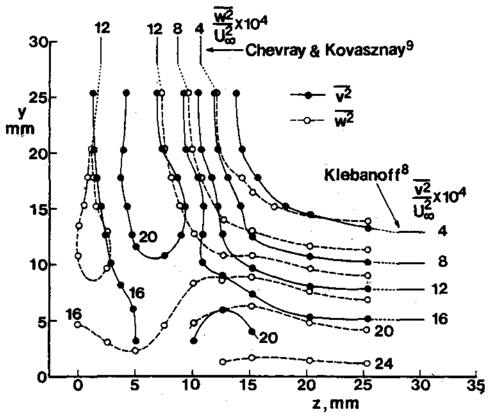
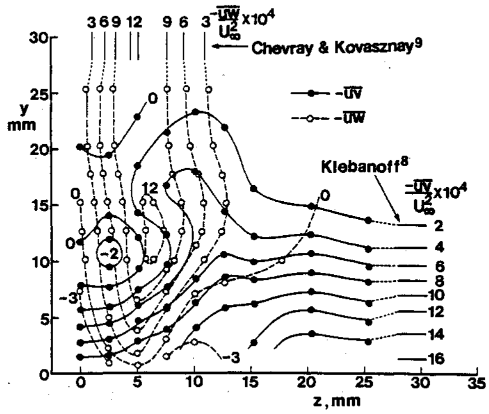


a) u^2/U_∞^2 at $x=25.4$ and 101.6 mm.



b) $\overline{v^2}/U_\infty^2$ and $\overline{w^2}/U_\infty^2$ at $x=101.6$ mm.



c) $-\overline{uv}/U_\infty^2$ and $-\overline{uw}/U_\infty^2$ at $x=101.6$ mm.

Fig. 3 Reynolds stress distributions.

Results and Discussion

The pressure distribution on the body was found to be very uniform within a scatter of 0.5% of the freestream dynamic pressure, except in the area within 5 cm from the root of the wing trailing edge, where the pressure was higher and the pressure coefficient was 0.03 at the root of the wing trailing edge. The symmetry of the flow about the centerplane ($z=0$) was verified at a few streamwise positions and most of the detailed data were obtained on one side ($z \geq 0$).

The development of the primary mean flow and secondary flow vectors is presented in Fig. 2. The shapes of the isovel contours at $x=0$ are very close to the results of Mojola and Young[1] taken in a 90-deg streamwise corner, except that in the present flow the Reynolds number is smaller and the contours near the surfaces are more closely spaced. The characteristics of the boundary layers at $x=0$ far away from

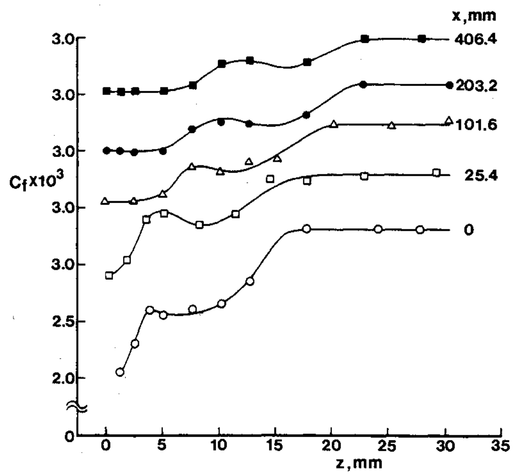


Fig. 4 Distribution of skin friction coefficient.

the corner are: at large z, the boundary-layer thickness $\delta_{0.995}=1.47$ cm, the momentum thickness $\theta=0.147$ cm, and the shape parameter $H=1.35$; and at large y, $\delta_{0.995}=1.32$ cm, $\theta=0.132$ cm, and $H=1.35$. The main feature of the mean flow is that the streamwise velocity first increases rapidly near the centerplane while in the rest of the regions the flow develops slowly, retaining the qualitative features of a corner flow. The acceleration near the centerplane, which is larger at larger y, directs the secondary flow motions upward and toward the centerplane. This shifted secondary flow motion is better seen at the downstream station in Fig. 2b.

The squared streamwise intensity $\overline{u^2}$ at $x=25.4$ and 101.6 mm is shown in Fig. 3a and the Reynolds stresses $\overline{v^2}$, $\overline{w^2}$, $-\overline{uv}$, and $-\overline{uw}$ at $x=101.6$ mm are shown in Figs. 3b and 3c, together with the flat-plate boundary-layer data of Klebanoff[8] and the flat-plate wake data of Chevray and Kovasznay.[9] First, the shear stresses $-\overline{uv}$ and $-\overline{uw}$ go to zero satisfactorily at large y and z, respectively. Second, the profiles of $\overline{u^2}$, $\overline{v^2}$, and $-\overline{uv}$ at large z agree well with the flat-plate data of Klebanoff[8] scaled up by the ratio of the present C_f to that of Klebanoff. The profiles of $\overline{u^2}$, $\overline{w^2}$, and $-\overline{uw}$ at large y also agree with the flat-plate wake data of Chevray and Kovasznay[9] interpolated at the value of $u_0^* x/\nu$ corresponding to the present flow, where u_0^* is the friction velocity of the boundary layer at the wing trailing edge. It should be noted that the symmetry condition $-\overline{uw}=0$ at $z=0$ is not quite satisfied, especially at small y. A general trend of the Reynolds stress distributions is that they all decrease near the centerplane. It is seen that in the region where the mean velocity field is changing rapidly (near $y=10$ mm, $z=3$ mm), the sign of $-\overline{uv}$ is negative, while $\partial U/\partial y$ is positive or almost zero. The extent of the region of negative eddy viscosity appears to be small. Figure 4 presents the distribution of the friction coefficient C_f as determined by Clauser's method of fitting the logarithmic portion of mean velocity profiles. C_f appears to be always smaller inside the wake-influenced region than in the asymptotic boundary layer at large z.

Acknowledgments

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Linearization of Turbulent Boundary-Layer Equations

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Introduction

IT is common practice to linearize the boundary-layer equations with Newton's method, which usually provides quadratic convergence. The linearization of the laminar-flow equations is straightforward but the expressions used to model the Reynolds stresses present difficulties when it is applied to the turbulent flow equations. Indeed, in previous solutions of the turbulent boundary-layer equations that use the eddy-viscosity concept in algebraic form, Newton's method has been applied to all terms in the equation except for the viscous term where values of the eddy viscosity were assumed from a previous iteration. In this way the linearized system of equations has been solved with prior knowledge of the terms that represent turbulent diffusion and with the consequence that solutions oscillate and the convergence rate is slower than it need be. In a typical airfoil calculation, about four iterations are required at each longitudinal station to achieve a 1% tolerance error in wall shear stress; with, say, 50 stations the additional calculation effort is clearly significant.

The calculation times that result from this incomplete application of Newton's method in the solution of the turbulent boundary-layer equations may be tolerated in situations where the freestream boundary condition is prescribed. However, where solutions of the inviscid- and viscous-flow equations are required to interact, several sweeps of the flowfield may be needed and the additional computer time becomes important. The method described and evaluated here was devised to allow the application of Newton's method to all the terms in the momentum equation. The approach makes use of the eddy-viscosity formulation of Cebeci and Smith[1] and the Mechul function of Ref. 2, which regards the dimensionless displacement thickness and wall shear of the eddy-viscosity formula as unknowns.

Basic Equations

The boundary-layer equations and their boundary conditions for two-dimensional incompressible laminar and turbulent flows are well known. With the concept of eddy viscosity ϵ_m, and with $b = 1 + \epsilon_m/\nu$, they can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{\mathrm{d}u_e}{\mathrm{d}x} + \nu\frac{\partial}{\partial y}\left(b\frac{\partial u}{\partial y}\right) \tag{2}$$

$$y=0 \qquad u=v=0; \qquad y\to\delta \qquad u\to u_e \tag{3}$$

The presence of the eddy viscosity ϵ_m in b requires a turbulence model, and the algebraic eddy-viscosity formulation of Cebeci and Smith is used here. According to this formulation, ϵ_m is defined by two separate formulas given by

$$\epsilon_m = \left\{0.4y\left[1-\exp\left(\frac{-y}{A}\right)\right]\right\}^2\frac{\partial u}{\partial y} \qquad 0\le y\le y_c \tag{4a}$$

$$= 0.0168\int_0^\infty (u_e - u)\,\mathrm{d}y \qquad y_c\le y\le\delta \tag{4b}$$

where

$$A = 26\nu u_\tau^{-1}N, \quad N = (1-11.8p^+)^{-1/2},$$

$$u_\tau = \left(\frac{\tau_w}{\rho}\right)^{1/2}, \quad p^+ = \frac{\nu u_e}{u_\tau^3}\frac{\mathrm{d}u_e}{\mathrm{d}x} \tag{5}$$

The condition used to compute y_c is the continuity of the eddy viscosity; from the wall outward (inner region) Eq. (4a) is applied until its value is equal to the one given for the outer region by Eq. (4b).

For external flows, it is convenient to solve Eqs. (1) and (2) when they are expressed in transformed variables, and the Falkner-Skan transformation is used for this purpose. The result, with $m = x/u_e\,\mathrm{d}u_e/\mathrm{d}x$, is

$$(bf'')' + \frac{m+1}{2}ff'' + m[1-(f')^2] = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \tag{6}$$

$$\eta=0, \qquad f'=f=0; \qquad \eta\to\eta_e, \qquad f'\to 1 \tag{7}$$

In terms of transformed variables, the b term in Eq. (6) can be written as

$$b = 1 + a_1 f''[1-\exp(-a_2N^{-1}f_w''^{1/2})]^2\lambda_1 + a_3(\eta_e - f_e)\lambda_2 \tag{8}$$

Here λ_1 and λ_2 are determined by the continuity of eddy-viscosity formulas with $\lambda_1 = 1$ and $\lambda_2 = 0$ in the inner region and $\lambda_1 = 0$ and $\lambda_2 = 1$ in the outer region. a_1, a_2, a_3, and N are defined by

$$a_1 = 0.16R_x^{1/2}\eta^2, \quad a_2 = (R_x^{1/2}/26)\eta, \quad a_3 = 0.0168R_x^{1/2}$$

$$a_4 = 11.8mR_x^{-1/4}, \quad N = [1 - a_4(f_w'')^{-3/2}]^{-1/2}, \quad R_x = u_e x/\nu \tag{9}$$

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